Solutions to Midterm 1

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1 (10 points)

1.1 Part (a)
The DFS forest is showed in Fig. 1 where pre and post numbers are written in the form [pre,post] beside each node.

![DFS forest diagram](image)

Fig. 1. The DFS forest.

1.2 Part (b)
The shortest path tree is showed in Fig. 2 where the distances are written next to each node.

2 (10 points)

2.1 Part (a)
Polynomial time. We only need to output all nodes with zero in-degrees, which takes $O(|V| + |E|)$ time.

2.2 Part (b)
Exponential time. We have proved in homework 2-4 that graphs can have exponentially many cycles. It takes the computer at least exponential time to output all of them.
2.3 Part (c)

Exponential time. There are \(n!\) many topological orderings for a graph with \(n\) nodes and no edges. Since \(n! > 2^n\) when \(n\) is large enough, we need at least exponential time to output all of them.

3 (10 points)

3.1 Part (a)

Just find the SCC with largest number of nodes. Please refer to section 3.4 of the book for the time complexity.

3.2 Part (b)

One way of doing this is brute force. For each \(s \in V\), we use the DFS starting from \(s\) on the graph \(G_R\). If all vertices are visited, then \(s\) is universally accessible. The time complexity is \(O(|V||V| + |E|)\).

We can also find all SCCs of the graph and reduce it into a DAG. If the DAG has only one sink, all nodes in it are universally accessible. Otherwise, there is no universally accessible node. The time complexity is the same as part (a).

4 Exercise 4.20

4.1 Part (a)

Let \(d_1 = BFS(G, s)\) and \(d_2 = BFS(G^R, t)\). For each \(e' = (u,v) \in E'\), we define \(d_{e'} = \min(d_1[v], d_1[u] + d_2[v] + 1)\), which is the shortest distance from \(s\) to \(t\) after adding \(e'\). Therefore, the \(e'\) minimizing \(d_{e'}\) is our solution. We can also do this problem by adding each \(e'\) to the graph and computing \(d_{e'}\) by BFS. You will lose one point for this less efficient algorithm.
4.2 Part (b)

The running times for the fast algorithm and slow one are $O(|V| + |E| + |E'|)$ and $O(|E'|(|V| + |E'|))$, respectively.