Adaptive filtering of mobile robot velocity from laser range data

Yuncong Chen

Department of Computer Science
University of California, San Diego

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iSee, the mobile platform for SLAM

- 3 degrees of freedom, can move laterally
- 100 msec/scan
- 240° coverage with 0.36° angular resolution
- 4 meters maximum range
Motivation

- **SLAM** (Simultaneous Localization and Mapping): both the map and the pose \((x,y,\theta)\) are hidden states.
- Velocity information is important in SLAM, as it provides a priori estimation of robot pose.
  - dead reckoning using odometry or accelerometer accumulates error (e.g. slippery), need observation feedback to correct the error.
  - landmark-based Kalman filter requires first extracting features from sensor data
  - scan matching finds the most likely rigid transformation that explains the change between consecutive frames, need to analyze global data, computationally expensive.
- Apply **linear adaptive filters** to find a **fast** and **robust** method to estimate the velocity directly from **raw range data**.
Suppose the motion is small during a sampling cycle. Linearize the measurement model

\[ d(x, y, \theta)_{t+1} \approx d(x, y, \theta)_t + \frac{\partial d}{\partial x} v_x(t) + \frac{\partial d}{\partial y} v_y(t) + \frac{\partial d}{\partial \theta} \omega(t) \]

Approximate the derivatives with forward differences

\[
\frac{\partial d_i}{\partial x} \bigg|_t \approx -\frac{\sin(\alpha_i + \beta_i(t))}{\sin(\beta_i(t))} \\
\frac{\partial d_i}{\partial y} \bigg|_t \approx \frac{\cos(\alpha_i + \beta_i(t))}{\sin(\beta_i(t))} \\
\frac{\partial d_i}{\partial \theta} \bigg|_t \approx \frac{1}{\tan(\beta_i(t))} d_i(t)
\]

where

\[
\alpha_i = \phi(i - i_{\text{center}}) \in (-\pi, \pi] \quad \beta_i(t) \approx \sin^{-1} \left( \frac{\phi k}{d_i(t)} \frac{d_{i+k}(t)}{d_{i+k}(t) - d_i(t)} \right) \in (0, \pi)
\]

can be estimated based on data local to the scan ray.
So we can formulate the velocity filtering problem as estimating the coefficients of this linear model.

- **input**: derivatives that can be computed from measurements of last timestep. *noisy.*

- **weights**: velocities that need to be estimated.

- **output**: changes in range. *noisy.*
The estimation error of a single ray

\[ e_i(t + 1) = d_i(t + 1) - d_i(t) - w(t)^T x_i(t), \quad i = 1, \ldots, N \]

The goal is to minimize the total error

\[ \mathcal{E}(t + 1) = \sum_i e_i^2(t + 1) \]

Update the weights along the steepest descent direction of the current mean square error

\[ w(t + 1) = w(t) + \mu \nabla_w \mathbb{E}\{\mathcal{E}^2(t + 1)\} \]

\[ \approx w(t) + \mu \sum_i e_i(t + 1) x_i(t) \]
LMS $v_x = -0.01$, $v_y = 0.01$, $\omega = 0.02$

- learning rate higher than 0.0002 results in divergence
- estimation error may be caused by high incident angle of rays, because $\partial d/\partial \omega$ is proportional to $\tan(\beta)$ which shows sudden surge at angles close to $\pi$. 
LMS $v_x = -0.01$, $v_y = 0.01$, $\omega = 0.02$, $\beta < 2.5$

- Limit the valid $\beta$ to be smaller than 2.5 ($143^\circ$)
- Error is significantly reduced
- Legit learning rate increases to 0.0015
LMS Switching $v_x = \pm 0.01, \, v_y = \pm 0.01, \, \omega = \pm 0.02$

Filter with higher learning rate switches faster.
Setting the gradient of the discounted total error

\[ \sum_{\tau=1}^{t} \lambda^{t-\tau} E^{2}(\tau) \]

to zero yields

\[ \sum_{\tau=1}^{t} \lambda^{t-\tau} \sum_{i \text{ valid}} (d_i(\tau) - d_i(\tau - 1))x_i(\tau - 1)_k \]

\[ = \sum_{l=1}^{3} w_l \left[ \sum_{\tau=1}^{t} \lambda^{t-\tau} \sum_{i \text{ valid}} x_i(\tau - 1)_l x_i(\tau - 1)_k \right] \]

which can be written as

\[ R_x(t) w(t) = Q_{dx}(t) \]
RLS $\nu_x = -0.01, \nu_y = 0.01, \omega = 0.02$

- $\lambda = 1$, the filter takes past measurements into account when making estimation, the result is therefore close to the Wiener solution for stationary process.
- $\lambda = 0$, the filter finds the optimal estimate based on current measurement only.
- RLS does not have the problem of divergence.
RLS $v_x = -0.01$, $v_y = 0.01$, $\omega = 0.02$, $\beta < 2.5$

- Limit the valid $\beta$ to be smaller than 2.5 (143°)
- Error is reduced
RLS switching $v_x = \pm 0.01$, $v_y = \pm 0.01$, $\omega = \pm 0.02$

If filter has long memory, it is difficult to catch up switchings.

If filter has no memory, it adapts to switchings instantly.
Zigzag motion, facing forward
Parallel motion, facing forward
Both LMS and RLS do a good job filtering stationary processes.

RLS filters with short memory is especially suited for non-stationary processes as it can adapt quickly, but it comes with a cost of computational complexity.

Velocity filtering using adaptive filters

- is fast because it uses only local data for each scan ray
- is robust because the estimate is typically overdetermined
- can provide estimation at high frequency, and complement with the use of more complex methods (scan matching, Kalman filtering) that is performed at a lower frequency.
- is error-prone when rays hit a wall at a high incident angle. Limiting the range of valid incident angle can reduce this error.