Universal source coding
and
the Online Bayes algorithm

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Outline

Combining experts in the log loss framework
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The online Bayes Algorithm
  Comparison to $\text{Hedge}(\eta)$
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The performance bound
  Comparison to \textit{Hedge}(\eta)
  Advantage over two part codes
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Comparison with Bayesian Statistics

Computational issues
The log-loss framework

- Algorithm $A$ predicts a sequence $c^1, c^2, \ldots, c^T$ over alphabet $\Sigma = \{1, 2, \ldots, k\}$
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- The prediction for the $c^t$th is a distribution over $\Sigma$: $p^t_A = \langle p^t_A(1), p^t_A(2), \ldots, p^t_A(k) \rangle$
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- The cumulative log loss, which we wish to minimize, is $L_A^T = - \sum_{t=1}^{T} \log p_A^t(c^t)$
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- \( \lceil L_A^T \rceil \) is the code length if \( A \) is combined with arithmetic coding.
The game

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Goal: minimize regret:

$$- \sum_{t=1}^{T} \log p^t_A(c^t) + \min_{i=1,\ldots,N} \left( - \sum_{t=1}^{T} \log p^t_i(c^t) \right)$$
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- **Total loss** of expert $i$

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L_i^t = - \sum_{s=1}^{t} \log p_i^s(c^s); \quad L_i^0 = 0
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$$w_i^1 \geq 0, \quad \sum_{i=1}^{n} w_i^1 = 1$$
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The **Hedge**($\eta$) Algorithm

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- **Probability:**
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  p_t^i = \frac{w_t^i}{\sum_{j=1}^n w_t^j}, \quad p_t^* = \frac{w_t}{\sum_{j=1}^n w_t^j}
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Cumulative loss vs. Final total weight

Total weight: $W^t = \sum_{i=1}^{N} w_i^t$
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**EQUALITY** not bound!
Simple Bound

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- Dividing by $T$ we get $\frac{L_A^T}{T} = \min_i \frac{L_i^T}{T} + \frac{\log N}{T}$
Upper bound on $\sum_{i=1}^{N} w_i^{T+1}$ for $\text{Hedge}(\eta)$

Lemma (upper bound)

For any sequence of loss vectors $\ell^1, \ldots, \ell^T$ we have

$$\ln \left( \sum_{i=1}^{N} w_i^{T+1} \right) \leq -(1 - e^{-\eta}) L_{\text{Hedge}(\eta)}.$$
Tuning $\eta$ as a function of $T$

- trivially $\min_i L_i \leq T$, yielding

$$L_{\text{Hedge}(\eta)} \leq \min_i L_i + \sqrt{2T \ln N} + \ln N$$
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- per iteration we get:

$$\frac{L_{\text{Hedge}(\eta)}}{T} \leq \min_i \frac{L_i}{T} + \sqrt{\frac{2 \ln N}{T}} + \frac{\ln N}{T}$$
Bound better than for two part codes

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- If we use Bayes predictor + arithmetic coding we get:
  $$L_A = -\log W^{T+1} \leq \log K \max_i \frac{1}{NK} e^{-L^T_i} = \log N + \min_i L^T_i$$

We don’t pay a penalty for copies.
More generally, the regret is smaller if many of the experts perform well.
Online Bayes Alg.

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▶ For number of mistakes - Bayesian method cannot be “fixed”. Requires variable learning rate.
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- Bayesian tricks:
  - **Conjugate priors**: A prior over a continuous domain whose functional form does not change with when updated. Number of parameters defining posterior is constant. Update rule translates into update of parameters. Parameters correspond to “sufficient statistics”. Exists for the family of exponential distributions.
  - **Markov Chain Monte Carlo**: Sample the posterior.
Computational Issues

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   ▶ Read the background material I put on the class twiki page (for class no. 5)